Physics 8660, Fall 2008
Homework 6, 11/13/2008, due 12/04/2008

1 Seasonal variation of ground temperature

In the flat-earth approximation, the temperature ψ at depth z can be described by a one-dimensional diffusion equation of the form

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{\kappa}\frac{\partial \psi}{\partial t},$$

where κ is the thermal diffusion constant. Suppose that the external temperature can be approximated by a sinusoidal variation of the form

$\psi[0, t] = \psi_0 + \psi_1 \text{Sin}[\omega t]$

where $\omega = 2\pi/T$ for period T.

a) Solve for $\psi[z,t]$ and determine the *penetration depth d* and the *phase delay* for propagation of thermal waves into the ground.

b) The penetration depth for annual variations is approximately 3 meters. Determine the phase delay for annual variations. Also determine the corresponding penetration depth and phase delay for daily variations. Discuss the separability of these periods.

c) Obtain $\psi[z,t]$ for combined annual and diurnal variations.

2. Scattering by a soft cylinder

Plane waves with wave number k are incident upon an infinitely long cylinder of radius R with the direction of propagation perpendicular to the axis of the cylinder. The surface of the cylinder is soft, such that the total wave (incident plus scattered) vanishes at R. Construct a formal expansion for the scattered field and then examine the leading term in the long-wavelength limit $k R \ll 1$.