Physics 8660, Fall 2008
Homework 4, 10/16/2008, due 10/30/2008

1. **Cylinder with opposite potentials on its end caps**

The curved surface of a cylinder of radius *a* is grounded while the end caps at z=±L/2 are maintained at opposite potentials $\psi(r,\theta,\pm L/2) = \pm V(r,\theta)$.

a) Develop an expansion for the electrostatic potential $\psi(r,\theta,z)$ within the cylinder and express the coefficients in terms of the appropriate integral over $V(r,\theta)$.

b) Determine the coefficients for the simple case $V(r,\theta) = V_0$ where V_0 is constant.

2. **Heating simple solids**

Suppose that a simple solid (brick, sphere, cylinder, etc.) with uniform initial temperature is immersed at time *t* = 0 in a heat bath. The temperature $\psi(\vec{r},t)$ within the material satisfies

$\frac{1}{\kappa}\frac{\partial \psi}{\partial t} = \nabla^2 \psi$, $\psi(\vec{r},0) = \psi_i$, $\psi(\vec{R},t \geq 0) = \psi_f$, where $\vec{R}$ is on the surface.

a) Determine the temperature distribution for positive times within a brick with $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.

b) Determine the temperature distribution for positive times within a sphere with $r \leq R$. What is the asymptotic time dependence of the central temperature?

c) Determine the temperature distribution for positive times within a cylinder with $r \leq R, 0 \leq z \leq L$.