Physics 8660, Fall 2008
Homework 3, 10/02/2008, due 10/16/2008

1. (H.W. Wyld, Mathematical Methods for Physics, Problem 3.8)
a) Use the method of eigenfunction expansion in rectangular coordinates to find the solution of Laplace's equation
$\nabla^2\psi = 0$
for the region inside the cube
$0 < x < a$, $0 < y < a$, $0 < z < a$,
which satisfies the boundary conditions:
$\psi(0, y, z) = u_1(y, z)$
$\psi(a, y, z) = u_2(y, z)$
$\psi(x, 0, z) = \psi(x, a, z) = \psi(x, y, 0) = \psi(x, y, a) = 0$

b) Apply this solution to find the electrostatic potential inside a hollow cube with conducting walls when the right and left sides of the cube are kept at constant potential V and the other sides are kept at zero potential. Evaluate the expansion coefficients explicitely.

c) Evaluate numerically the potential at the center of the cube. How many terms of the series are necessary to obtain a result to three significant figures?

d) find the solution of Laplace's equation for the inside of the cube which satisfies nonzero boundary conditions on all six sides:
$\psi(0, y, z) = u_1(y, z)$
$\psi(a, y, z) = u_2(y, z)$
$\psi(x, 0, z) = v_1(x, z)$
$\psi(x, a, z) = v_2(x, z)$
$\psi(x, y, 0) = w_1(x, y)$
$\psi(x, y, a) = w_2(x, y)$

2. (H.W. Wyld, Mathematical Methods for Physics, Problem 3.7)
The z-independent solutions of Laplace's equation in cylindrical coordinates are

$$\psi = \left(A_0 + B_0 \ln r\right)\left(C_0 + D_0\theta\right) + \sum_{m=1}^{\infty}\left(A_m r^m + B_m r^{-m}\right)\left(C_m e^{im\theta} + D_m e^{-im\theta}\right).$$

Study the (z-independent) interior problem for the cylinder. Find the finite solutions which satisfies the boundary condition
$\psi(a, \theta) = u(\theta)$.
Sum the series and show that your solution can be written in the form:

$$\psi(r, \theta) = \frac{a^2 - r^2}{2\pi}\int_0^{2\pi} d\theta' u(\theta)\frac{1}{a^2 + r^2 - 2ar\cos(\theta - \theta')}.$$