Physics 8660, Fall 2008
Homework 2, 09/18/2008, due 10/02/2008

1. a) Show that the equation

$$u'' + \frac{1}{4x^2}(1 - x^2)u = 0$$

has two solutions of the form

$$u_1 = x^{\frac{1}{2}}\left(1 + \frac{x^2}{16} + \frac{x^4}{1024} + \cdots\right),$$

$$u_2 = u_1(x)\ln x - \frac{x^{\frac{5}{2}}}{16} + \cdots$$

near x=0.

b) Use Mathematica to find the solution u_1. Start from the Mathematica template that we used in class and modify it to work for the above equation. Add a printout of your script.

2. Find the eigenfunctions $u_n(x)$and eigenvalues λ_n for the differential equation $\frac{d^2u_n(x)}{dx^2} = -\lambda_n u_n(x)$ in the interval $0 \le x \le a$ for the following sets of boundary conditions:

a) $u(0) = 0$ and $u(a) = 0$.

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a) $u(0) + au'(0) = 0$ and $u(a) - au'(a) = 0$. Find the equation which determines the eigenvalues and verify that there is an infinite set of eigenfunctions and eigenvalues.