Physics 8660, Fall 2008
Homework 1, 09/04/2008, due 09/18/2008

1. Develop the Laplacian operator in spherical coordinates:

$$\boxed{\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}}$$

2. a) Consider the wave equation in one dimension

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0\,.$$

Change to variables $\xi = x - ct$ and $\eta = x + ct$
And show that the wave equation assumes the form

$$\frac{\partial^2 u}{\partial\xi\partial\eta} = 0$$

and that this has the general solution
$u = f(\xi) + g(\eta) = f(x-ct) + g(x+ct)$, where f and g are arbitrary functions.

b) Interpret the two contributions f(x-ct) and g(x+ct) and justify the statement often made in the text that c is the velocity of the waves. Draw some pictures to explain your considerations.

c) Suppose that u and $\frac{\partial u}{\partial t}$ are given at t=0:

$$u(x,0) = U(x)\,,$$

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = V(x)\,.$$

Show that these conditions determine the functions f and g:

$$f(x) = \frac{1}{2}U(x) - \frac{1}{2c}\int_a^x dx'V(x')\,,$$

$$g(x) = \frac{1}{2}U(x) + \frac{1}{2c}\int_a^x dx'V(x')\,,$$

so that

$$u(x,t) = f(\xi) + g(\eta) = f(x-ct) + g(x+ct)$$

$$= \frac{1}{2}U(x-ct) - \frac{1}{2c}\int_a^{x-ct} dx'V(x') + \frac{1}{2}U(x+ct) + \frac{1}{2c}\int_a^{x+ct} dx'V(x')$$